Decay of de Sitter vacua by thermal activation

Jaume Garriga[1,2] and Ariel Megevand[1,2]

[1] *Departament de Física Fonamental, Universitat de Barcelona,*
Diagonal 647, 08028 Barcelona, Spain and

[2] *IFAE, Campus UAB, 08193 Bellaterra (Barcelona), Spain*
(Dated: February 1, 2008)

Decay of a de Sitter vacuum may proceed through a "static" instanton, representing pair creation of critical bubbles separated by a distance comparable to the Hubble radius – a process somewhat analogous to thermal activation in flat space. We compare this with related processes recently discussed in the literature.

I. INTRODUCTION

In field theory, a metastable false vacuum may decay either by tunneling or by thermal activation. Tunneling is described by a solution of the Euclidean field equations symmetric under spacetime rotations [1]. In flat space and at zero temperature this is the instanton with the least action, and hence represents the dominant contribution to the decay rate [2]. At sufficiently high temperature, thermal activation is more probable than tunneling, and the rate is dominated by a static solution which represents a spherical critical bubble in unstable equilibrium between expansion and collapse [3]. The symmetry of this solution is O(3)× U(1), where the U(1) factor corresponds to translations along the compactified Euclidean time direction.

As shown by Coleman and de Luccia [4], gravity can be easily incorporated into the description of tunneling. When the initial state is a false vacuum with a positive energy density, the initial geometry corresponds to a de Sitter-like exponential expansion. A bubble which materializes through quantum tunneling has zero energy, and consequently (in the thin wall limit) the geometry outside of the bubble remains unaffected by the nucleation event. After nucleation, the bubble walls accelerate outward, and the volume of the new phase increases at the expense of the old one. However, due to the presence of event horizons in de Sitter, the growth of a single bubble cannot engulf the whole space. If the nucleation rate per unit volume Γ is large compared to H^4, where H is the inverse de Sitter radius, the foam of nucleated bubbles will percolate and the phase transition will complete. But if $\Gamma \ll H^4$, the rate at which bubbles nucleate and grow does not catch up with the exponential expansion of the false vacuum. In this case, the volume of the false vacuum keeps increasing with time and the transition never fully completes in the whole spacetime (leading to eternal old inflation). However, any observer will experience a local transition to the new vacuum phase in a finite proper time.

On the other hand, the description of thermal activation can be more involved when the self-gravity of the bubbles and of the thermal bath is considered. In the cosmological context, the thermal bath drives the expansion of the universe, and the temperature becomes time dependent. Because of that, exact instantons cannot be constructed. In low energy cosmological phase transitions (e.g. at the electroweak scale) it is safe to ignore the self-

gravity of the bubbles, and to use the flat spacetime results for the nucleation rate[1]. If this rate is sufficiently large, the phase transition completes as the bubbles percolate, typically after a short period of supercooling and subsequent release of latent heat by the nucleated bubbles [6]. But if the rate is too small, there is a strong supercooling and the thermal bath dilutes away; the false vacuum starts dominating before the phase transition is complete, and we are back to the situation described at the end of the previous paragraph [7].

Note, however, that even when all matter has been diluted away, the false vacuum dominated de Sitter expansion can be considered to have a non-vanishing temperature [8] $T = H/2\pi$, which does not dilute further as the universe expands. One may then ask whether this leads to thermal activation, and if so, at what rate does it proceed. In what follows, we shall discuss the corresponding instanton, describing the nucleation of a pair of critical bubbles in unstable equilibrium between expansion and collapse. It should be noted that the theorem in Ref. [2] does not necessarily apply to de Sitter space, and hence it is not clear a priori whether this new instanton has higher action than the usual Coleman-de Luccia one.

Static self-gravitating instantons with O(3) symmetry have previously been considered in a variety of contexts, notably for the description of false vacuum decay in the presence of a black hole (See e.g. [9, 10, 11] and references therein). The particular solution we shall consider here corresponds to pair creation of critical bubbles in de Sitter, and to our knowledge it does not seem to have received much attention in the past. The following is an extended version of the discussion given by the present authors in [12]. In Section II we describe the solution. In Section III we discuss the action and the nucleation rate. In Section IV we consider the limit in which the mass of the nucleated bubbles is small. Section V is devoted to the opposite limit, when the gravity of the bubbles is very important. In Section VI we compare the action for thermal activation with the action for tunneling (through the Coleman-de Luccia instanton). Section VII compares the process of thermal activation of seeds of the new phase with a related process recently discussed by Gomberoff et al. [13], by which most of space would jump to the new phase except for a pair of bubbles which contain the "remnant" of the old phase. Section VIII is devoted to conclusions.

II.   PAIR CREATION OF CRITICAL BUBBLES

Unlike the case of the Coleman-de Luccia bubble, the energy of a critical bubble is different from zero, and consequently, the metric outside of it is no longer pure de Sitter but Schwarzschild-de Sitter (SdS). The instanton is a solution of the Euclidean equations of motion, with two metrics glued together at the locus of the wall, which is a surface of constant r in the static chart of SdS (see Fig. 1). For simplicity, we shall restrict attention to the case where the vacuum energy density is positive in both the initial and the final states. Also, we shall assume that the thin wall approximation is valid [1].

---

[1] The effects of self-gravity, however, may be important at high energy scales, see e.g. [5] for a recent discussion.

A. The instanton

The metric outside is given by

$$ds^2 = f_o(r) \, dt^2 + f_o^{-1}(r) \, dr^2 + r^2 d\Omega^2, \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, and

$$f_o(r) = \left(1 - \frac{2GM}{r} - H_o^2 r^2\right). \tag{2}$$

The metric inside is given by

$$ds^2 = C^2 f_i(r) \, dt^2 + f_i^{-1}(r) \, dr^2 + r^2 d\Omega^2, \tag{3}$$

where

$$f_i(r) = \left(1 - H_i^2 r^2\right). \tag{4}$$

Here G is Newton's constant. The parameter C is determined by the condition that on the bubble wall (i.e., at $r = R$) the two metrics must agree, which leads to $C = [f_o(R)/f_i(R)]^{1/2}$.
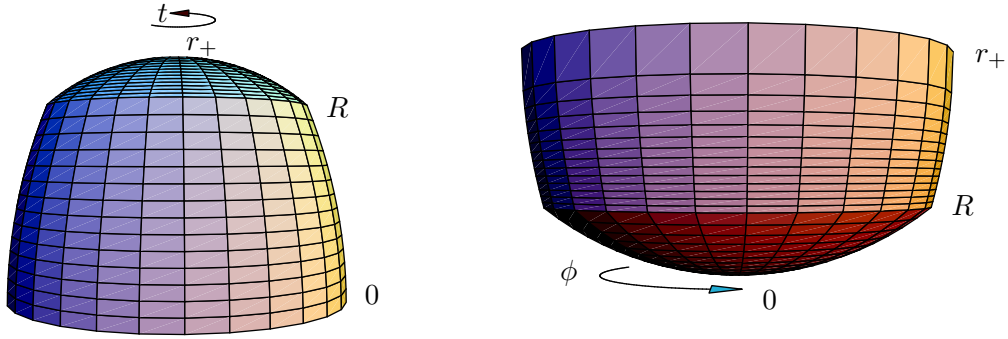

FIG. 1: Static instanton in de Sitter space. The left figure shows the geometry induced on the plane r, t, while keeping angular coordinates fixed, whereas the right figure shows the geometry induced on the plane r, ϕ, keeping θ and t fixed. The vertical direction corresponds to the coordinate r, common to both pictures. The cosmological horizon is at $r = r_+$, the buble wall is at $r = R$, and $r = 0$ is the center of the static bubble of the new phase. The geometry at the time of nucleation is obtained by cutting the instanton by a smooth spacelike surface orthogonal to the time-like killing vector. This corresponds to a diametral section of the figure on the left, which therefore contains two bubbles, whose centers are separated by a distance comparable to the Hubble radius.

The parameters M and R depend on the wall tension σ, and the Hubble parameters outside and inside the bubble, H_o and H_i respectively. Their values are determined by the junction conditions at the bubble wall [14],

$$[K_{ab}] = -4\pi G \sigma \gamma_{ab}, \tag{5}$$

where $[K_{ab}]$ is the difference in the extrinsic curvature $K_{ab} = (1/2)f^{1/2}\partial_r g_{ab}$ on the two sides and γ_{ab} is the world-sheet metric. Eq. (5) gives rise to the junction conditions,

$$[g] = -4\pi G \sigma, \quad [g'] = 0, \tag{6}$$

where we have introduced the new function $g(r) = f^{1/2}(r)/r$. Using Eqs. (2) and (4), we have

$$g_o g_o' = -\frac{1}{r^3} + \frac{3GM}{r^4}, \qquad g_i g_i' = -\frac{1}{r^3}. \tag{7}$$

From (6), we have $g_o'(R) = g_i'(R) = -3M/4\pi\sigma R^4$, and then $g_i(R)$ and $g_o(R)$ are easily obtained from Eqs. (7):

$$g_i(R) = \frac{4\pi\sigma R}{3M}, \qquad g_o(R) = g_i(R)\left(1 - \frac{3GM}{R}\right). \tag{8}$$

From (2) and (4) we have

$$g_o^2(R) = \frac{1}{R^2} - \frac{2GM}{R^3} - H_o^2, \qquad g_i^2(R) = \frac{1}{R^2} - H_i^2. \tag{9}$$

Inserting (8) in (9) we finally obtain a quadratic equation for $g_i(R) \equiv x$. The solution is

$$x = \frac{\epsilon}{4\sigma} + \frac{3\sigma}{16M_p^2} + \left[\left(\frac{\epsilon}{4\sigma} + \frac{3\sigma}{16M_p^2}\right)^2 + \frac{H_i^2}{2}\right]^{1/2}, \tag{10}$$

where we have introduced the parameter ϵ representing the difference in vacuum energy difference on both sides of the bubble wall: $H_o^2 - H_i^2 = 8\pi G\epsilon/3 = \epsilon/3M_p^2$. Then the parameters M and R are given in terms of x by

$$R^{-2} = x^2 + H_i^2, \qquad M = 4\pi\sigma R/3x. \tag{11}$$

This concludes the construction of the instanton solution for given values of the physical parameters σ, H_o and H_i.

B. Cosmological horizon

The above equations are valid only as long as $3GM < R$ [otherwise (8) would yield $g_i < 0$, which is meaningless]. Thus, from (11), we require $4\pi G\sigma < x$, or $\sigma < \sigma_N$, where

$$\sigma_N^2 = 4M_p^4(3H_o^2 - H_i^2). \tag{12}$$

(The case with $\sigma > \sigma_N$ will be discussed in Section V.) The mass parameter satisfies

$$M \leq M_N \equiv M(\sigma_N) = (3\sqrt{3}GH_o)^{-1}. \tag{13}$$

For $M < M_N$, the equation $f_o(r) = 0$ has three real solutions. One of them, say r_-, is negative and the other two are positive. The two positive roots correspond to the black hole and cosmological horizons. We call them respectively r_s and r_+. Therefore we can write

$$f_o(r) = -\frac{H_o^2}{r}(r - r_-)(r - r_s)(r - r_+). \tag{14}$$

In the present case, with $\sigma < \sigma_N$, the horizon at r_s is not present, since the exterior metric is matched to an interior metric at some $r = R > r_s$ (see Fig. 1). For $r < R$ the metric is

just a ball of de Sitter in the static chart, and it is regular down to the center of symmetry at $r = 0$. In general, the size of the cosmological horizon is given by

$$r_+ = \frac{2H_o^{-1}}{\sqrt{3}} \cos\left(\frac{\varphi + \pi}{3}\right),\tag{15}$$

where we have introduced the angle

$$\varphi = -\arctan\sqrt{\frac{1}{27H_o^2 M^2 G^2} - 1},\tag{16}$$

In the limit $M \to 0$ the angle $\varphi \to -\pi/2$, and $H_o r_+ \to 1$.

According to Eq (8), on the bubble wall we have $f_o(R) = x^2 (R - 3GM)^2$, so the equation $f_o(R) = 0$ has a double zero instead of two different roots. This means that the radius of the instanton will coincide with the radius of one of the horizons only in the special case where both horizons have the same size, $r_s = r_+ = R = 3GM$. As we shall see in Section V, this limit corresponds to $\sigma = \sigma_N$, for which the exterior metric is the Nariai solution [15, 16], with mass parameter M_N given in (13) and with $H_o r_+ \to 1/\sqrt{3}$. There, we shall also comment on the case $\sigma > \sigma_N$, which is not covered by the present discussion.

C. Euclidean periodicity

Regularity of the Euclidean solution will determine the periodicity of the Euclidean time coordinate (and the thermal properties of the solution). For $r \to r_+$, we have

$$f_o(r) \approx A^2 \left(1 - \frac{r}{r_+}\right),\tag{17}$$

where

$$A^2 = H_o^2 (r_+ - r_-)(r_+ - r_s) = 3H_o^2 r_+^2 - 1.\tag{18}$$

In terms of the new coordinates

$$\rho = \frac{2r_+}{A}\sqrt{1 - \frac{r}{r_+}}, \quad \phi = \frac{A^2}{2r_+}t,\tag{19}$$

the metric (1) for $r \to r_+$ reads

$$ds^2 = \rho^2 d\phi^2 + d\rho^2 + r_+^2 d\Omega^2,\tag{20}$$

so it is clear that ϕ is an angle, $0 \le \phi \le 2\pi$, and t varies in the range $0 \le t \le 4\pi r_+/A^2$. Thus, the periodicity of the time coordinate is given by

$$\beta = \frac{4\pi r_+}{3H_o^2 r_+^2 - 1} = \frac{2\pi r_+^2}{r_+ - 3GM}.\tag{21}$$

It is also of some interest to determine the physical temperature on the bubble wall world-sheet, given by the proper time periodicity $\beta_R \equiv \int_0^\beta f_o^{1/2}(R)\, dt = f_o^{1/2}(R)\, \beta = C f_i^{1/2}(R)\, \beta$,

$$\beta_R = 2\pi x r_+^2 \frac{R - 3GM}{r_+ - 3GM}.\tag{22}$$

If there are field degrees of freedom living on this worldsheet, this will be the temperature that they will experience (rather than the ambient de Sitter temperatures).

Like in the case of instantons describing the production of black holes [15] or monopoles [17] in de Sitter, the instanton presented here describes the creation of *pairs* of bubbles. As we have just seen, the Euclidean time runs on a circle S^1 (See Fig. 1). The geometry at the time of nucleation is obtained by slicing the compact instanton through a smooth spacelike surface which cuts the S^1 factor at two places, say, $t = 0$ and $t = \beta/2$. The resulting geometry contains two different bubbles separated by a distance comparable to the inverse expansion rate.

III.   INSTANTON ACTION

The nucleation rate is determined by the Euclidean action, which turns out to have a rather simple expression in terms of r_+. The action is given by

$$S_E = \sigma \int d^3\xi \sqrt{\gamma} + \int d^4x \sqrt{g} \left(\rho_V - \frac{\mathcal{R}}{16\pi G} \right). \tag{23}$$

By the equations of motion, the scalar curvature is given by

$$\mathcal{R}\sqrt{g} = 32\pi G \rho_V \sqrt{g} + 24\pi G \sigma \int d^3\xi \sqrt{\gamma} \delta^{(4)} \left(x - x\left(\xi \right) \right), \tag{24}$$

and hence the on shell action is given by

$$S_E = -\frac{\sigma}{2} \int d^3\xi \sqrt{\gamma} - \int d^4x \rho_V \sqrt{g}. \tag{25}$$

The first integral in (25) is just the volume of a two-sphere of radius R times β_R. The second integral in (25) splits into the contributions from the two different vacua,

$$\rho_i \int_0^R C dt dr 4\pi r^2 + \rho_o \int_R^{r_+} dt dr 4\pi r^2 \tag{26}$$

$$= \rho_i C \beta \frac{4}{3}\pi R^3 + \rho_o \beta \frac{4}{3}\pi \left(r_+^3 - R^3 \right) \tag{27}$$

So the instanton action is

$$S_E = -2\pi R^2 \sigma f_o^{1/2}\left(R \right) \beta - R^3 \frac{H_i^2}{2G} \frac{f_o^{1/2}\left(R \right)}{f_i^{1/2}\left(R \right)} \beta - \left(r_+^3 - R^3 \right) \frac{H_o^2}{2G} \beta. \tag{28}$$

After some algebra S_E can be written in the simple form

$$S_E = -\frac{\pi r_+^2}{G} = -\frac{A(r_+)}{4G}, \tag{29}$$

where $A(r_+)$ is the area of the horizon at r_+. The exponent B which gives the probability for brane nucleation is the difference in actions between instanton and background. The

action of the background is just $S_E = -\pi/GH_o^2$, so the difference in actions between the instanton and the background is given by

$$B = \frac{\pi}{GH_o^2} \left(1 - r_+^2 H_o^2\right). \tag{30}$$

This expression could have been anticipated from general arguments. The action of a static Euclidean solution with periodicity β is given by [18, 19] $S_E = \beta E - S$, where E is the *total* energy and S is the entropy. For a closed solution, without a boundary, the total energy vanishes $E = 0$. Hence, the Euclidean action is equal to the entropy of the solution, which is just one fourth of the sum of the areas of all horizons. Hence, Eq. (30) can be rewritten as $B = -\Delta S$, and the transition probability is proportional to

$$\Gamma \sim \exp(\Delta S). \tag{31}$$

As both the initial and the final solutions can be said to represent macroscopic states in thermal equilibrium, their entropy can be interpreted as the logarithm of the corresponding number of microstates in the microcanonical ensemble with $E = 0$. The transition probability is then simply proportional to the relative number of microstates.

IV.   WEAK SELF-GRAVITY LIMIT

If the bubbles are sufficiently light, $GMH_o \ll 1$, and provided that $\sigma < \sigma_N$, Eq. (16) gives $H_o r_+ \approx 1 - GMH_o$, and from (30) we have

$$B \approx \beta_o M, \tag{32}$$

where $\beta_o \equiv 2\pi/H_o$. Since the mass of the bubble is small, its appearance does not significantly change the temperature of the horizon. In such case, the nucleation rate of the bubbles may be interpreted from the point of view of the observer at $r = 0$ as being due to a thermal bath at the fixed Gibbons-Hawking temperature β_0^{-1}. The corresponding probability is proportional to the Boltzman factor

$$e^{-\beta_0 M}. \tag{33}$$

Eq. (32) can also be understood as follows. The energy of the bubble has been extracted from thermal reservoir. According to the first law, the entropy of the reservoir must decrease by $dS_{horizon} = -\beta_0 \, dM$ [20]. From the discussion at the end of the last Section, $B = -(\Delta S)_{horizon}$, and hence for bubbles of small mass we obtain (32).

Let us now consider a few specific limiting cases, starting with the case of low tension branes, $\sigma/M_p^2 \ll H_o, H_o - H_i$, with $H_o > H_i$. In this case the parameter x is large compared with H_o, $R \simeq x^{-1}$, and we have

$$M = 16\pi\sigma^3/3\epsilon^2.$$

This is just the flat space expression for the energy of a critical bubble. The corresponding bounce action is $B \approx 32\pi^2\sigma^3/3H_o\epsilon^2$, which coincides with the thermal activation rate in flat space at the temperature β_o^{-1}.

At finite temperature, jumps to a vacuum with a higher energy density are also possible. In the absence of gravity, these jumps are frustrated because the bubbles of the new phase tend to recollapse. When gravity is included, the expansion of the universe can keep these

"false vacuum" bubbles from contracting (this is true also for the case of tunneling bubbles [21]). Hence, let us consider again the case of low tension branes, $\sigma/M_p^2 \ll |H_o - H_i| \ll H_i$, but now with $H_o < H_i$. In this case we find $x \approx -\sigma H_i^2/\epsilon \ll H_i$, and

$$R \approx H_i^{-1} \left[1 - \frac{1}{2} \left(\frac{H_i \sigma}{\epsilon} \right)^2 \right] \approx H_i^{-1}.$$

In this limit, the bubble of the false vacuum phase is almost as big as the cosmological horizon. We also have $M \approx -(4\pi/3)\epsilon H_i^{-3}$ and $B \approx -8\pi^2\epsilon/3H_o^4$, where we have used that $H_i - H_o \ll H_i$ to replace H_i by H_o in the last expression (note that $\epsilon < 0$ in the case we are considering here, so M and B are both positive). This approximately coincides with the bounce action for the homogeneous Hawking-Moss instanton [22], representing the upward jump of a horizon sized region of de Sitter space into a higher false vacuum.

Finally, we may consider the case of intermediate tension $|H_o - H_i| \ll \sigma/M_p^2 \ll H_o, H_i$. This leads to $x \approx H_i/\sqrt{2}$, and $R^2 \approx 2/3H_i^2$. In this case, the difference in pressure between inside and outside of the brane is insignificant compared with the brane tension term, which is balanced against collapse by the cosmological expansion. The energy of the critical bubble is $E_c(R) \approx 4\pi\sigma R^2$. Note, from (22), that the inverse temperature

$$\beta_R \approx \frac{2\pi}{\sqrt{3}H_o}, \tag{34}$$

is different from the one experienced by a geodesic observer at the origin of coordinates $r = 0$. This is because observers at $r \neq 0$ are in fact accelerating. From the point of view of the observer at $r = 0$, the energy of the bubble is $M = f_0^{1/2}(R)E_c(R)$, because of the gravitational potential contribution. Hence, taking into account that $\beta_R = f_0^{1/2}(R)\beta_0$ the exponent in the Boltzmann suppression factor can be written as $B \approx \beta_0 M \approx \beta_R E_c$, and we have $B \approx 16\pi^2\sigma/3\sqrt{3}H_o^3$.

V. STRONG GRAVITY LIMIT

For given H_o and H_i, the solution of Section II only exists provided that the tension of the bubble wall does not exceed a certain bound σ_N, given in Eq. (12). Let us now consider what happens near this bound, and beyond.

A. The Nariai limit

As we mentioned in the discussion below Eq. (14), the exterior metric in the limit $\sigma \to \sigma_N$ corresponds to the Nariai solution, with $r_s = r_+ = \left(\sqrt{3}H_o\right)^{-1}$, and $M = 1/3\sqrt{3}H_oG$. Replacing this value in (30) we find readily

$$B = \frac{2\pi}{3GH_o^2}. \tag{35}$$

This may be compared with the action of the instanton describing the nucleation of black holes in the same de Sitter universe [15],

$$B_N = \frac{\pi}{3GH_o^2}. \tag{36}$$

The difference $B - B_N = \pi/3GH_o^2 = A_{bh}/4G$, is just the area of the black hole horizon in the Nariai solution, as expected from the general discussion of the previous Section (Here, we are of course neglecting the entropy stored in the field degrees of freedom living on the bubble walls, which would show up when the determinantal prefactor in the nucleation rate is evaluated).

The fact that $r_s = r_+$ does not mean that both horizons coincide, since the coordinates r, t become inadequate in this case [15]. Near the Nariai limit the metric outside takes the form (1), with

$$f_o(r) \approx A^2 \left(1 - \frac{r}{r_+} \right) - \left(1 - \frac{r}{r_+} \right)^2, \tag{37}$$

and $r \approx r_+$, plus higher orders in the parameter A, which we defined in (18). In the present limit this parameter tends to zero, $A^2 = \sqrt{3}H_o \left(r_+ - r_s \right)$. Now we define new coordinates ψ and λ by

$$\cos \psi = 1 - \frac{2}{A^2} \left(1 - \frac{r}{r_+} \right), \quad \lambda = \frac{A^2}{2} t, \tag{38}$$

so that the metric becomes

$$ds^2 = \sin^2 \psi \, d\lambda^2 + r_+^2 d\psi^2 + r_+^2 d\Omega^2. \tag{39}$$

The cosmological horizon is at $\psi = 0$ and the black hole horizon is at $\psi = \pi$. Now in the limit $A \to 0$ we just replace $r_+ = \left(\sqrt{3}H_o \right)^{-1}$.

We must determine the position ψ_R of the bubble wall, which is given as before by the matching conditions (5), where now the metric outside is (39). So, on the wall, we have

$$ds_\sigma^2 = \sin^2 \psi_R \, d\lambda^2 + r_+^2 d\Omega^2 \tag{40}$$

$$= f_i(R) \, dt'^2 + R^2 d\Omega^2. \tag{41}$$

The extrinsic curvature on the outside is $-(1/2)\partial_\psi g_{ab}$, with $g_{00} = \sin^2 \psi$ and $g_{\Omega\Omega} = r_+^2$, i.e., $K_{00} = -(1/r_+)g_{00} \cot \psi, K_{\Omega\Omega} = 0$. The curvature inside is as before $K_{00} = g_{00}\partial_r f_i^{1/2}$ and $K_{\Omega\Omega} = g_{\Omega\Omega}f_i^{1/2}/r$, with $f_i(r) = (1 - H_i^2 r^2)$, so the Israel conditions give

$$-\frac{1}{r_+} \cot \psi_R - \left(f_i^{1/2} \right)' |_R = -4\pi G\sigma, \tag{42}$$

$$f_i^{1/2}(R)/R = 4\pi G\sigma. \tag{43}$$

These equations are easily solved and give

$$\sin \psi_R = \left(\frac{3H_o^2 - H_i^2}{6H_o^2 - H_i^2} \right)^{1/2}, \tag{44}$$

$$\sigma = \sigma_N = 2M_p^2 \sqrt{3H_o^2 - H_i^2} \tag{45}$$

so H_i must be less than $\sqrt{3}H_o$. Now regularity at the cosmological horizon $\psi \simeq 0$ implies that $0 \leq \lambda/r_+ \leq 2\pi$, so $\beta_R = \sin(\psi_R) 2\pi r_+$. Hence,

$$\beta_R = \frac{2\pi}{\sqrt{3}H_o} \left(\frac{3H_o^2 - H_i^2}{6H_o^2 - H_i^2} \right)^{1/2}. \tag{46}$$

Thus, also in this case, the effective temperature of the field degrees of freedom living on the worldsheet will be of order H_0 (The only exception occurs if there is some fine adjustment between H_o and H_i which makes the factor inside the brackets very small, in which case the temperature may be much larger.)

B.   Beyond the Nariai limit

For $\sigma > \sigma_N$ we have $3GM > R$ and the construction of Section II does not apply [since by Eq. (8), g_i would be negative]. As pointed out in [13], above this threshold a static solution can still be constructed by gluing the interior solution (3) to the $r_s < r < R$ portion of the exterior SdS solution (1) (rather than using the $R < r < r_+$ portion). This changes the sign of K_{ab} in the exterior, and the junction conditions become

$$\{g\} = 4\pi G\sigma, \qquad \{g'\} = 0, \tag{47}$$

where the curly brackets denote twice the average value on both sides of the bubble wall. Eq. (8) is then replaced by

$$g_i\left(R\right) = \frac{4\pi\sigma R}{3M}, \qquad g_o\left(R\right) = g_i\left(R\right)\left(\frac{3GM}{R} - 1\right), \tag{48}$$

but Eqs. (10) through (11) remain the same. The instanton would still look pretty much as in Fig. 1, but with the cosmological Horizon of radius r_+ replaced by a black hole horizon of radius $r_s < R$. Hence, in the right pannel of Fig. 1, the horizontal maximal circles would grow from 0 to R as we move up from the center of the bubble, but then the circles would start decreasing from R to r_s as we continue from the bubble wall to the horizon.

It is straightforward to calculate the Euclidean action for this solution, which is given by

$$S_E = -\frac{A(r_s)}{4G}, \tag{49}$$

where $A(r_s) = 4\pi r_s^2$ is the area of the black hole horizon, with

$$r_s = \frac{2H_o^{-1}}{\sqrt{3}}\cos\left(\frac{\varphi - \pi}{3}\right),$$

and where φ is given by (16). The corresponding bounce action

$$B = \frac{\pi}{GH_o^2}\left(1 - r_s^2 H_o^2\right), \tag{50}$$

is perfectly finite, since the instantons involved are both compact and regular. Moreover, $B > 0$, as it should be if this is to be interpreted as a process with an exponentially suppressed rate.

Is this instanton suitable for describing vacuum decay in the usual sense? Let us assume that we are in a false vacuum phase, and for simplicity, that the false vacuum decay rate per unit volume is exceedingly small compared with H^4. Then we expect that after some time the metric will take the form

$$ds^2 = -d\tilde{t}^2 + e^{2H_o\tilde{t}}(d\vec{x})^2, \tag{51}$$

over an exponentially large portion of space (with the exception of small portions of volume carved out by bubbles of the new phase which may have nucleated). In the solution described in Section II, an asymptotic region with metric (51) can be found (upon analytic continuation) in the region beyond the cosmological horizon, which is asymptotically de Sitter and infinite in volume in a flat slicing. In the case we are considering in this subsection,
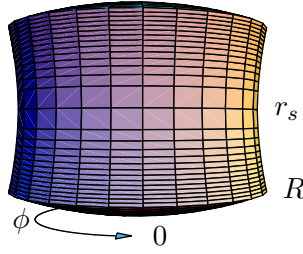
FIG. 2: The $t = const.$ surface has the geometry of an Einstein-Rosen bridge of the old phase which connects a pair of bubbles of the new phase.

however, the global structure of the solution is rather different. A black hole singularity is hidden beyond r_s, and the static solution does not contain any asymptotic region that looks like (51).

The $t = const.$ surface has the geometry of an Einstein-Rosen bridge of the old phase which connects a pair of bubbles of the new phase (see Fig. 2). Each one of these bubbles is in unstable equilibrium, it can either expand or contract. Let us concentrate in one of them. If the bubble wall expands, then its motion is perceived as expansion from both sides of the wall (recall that in the present case the radius r decreases as we move away from the wall in both directions). Conversely, contraction of the bubble wall would be perceived as contraction from both sides of the bubble wall. If one of the bubbles expands, it eventually generates an infinitely large region of the false vacuum phase surounding the black hole, and the metric in the false vacuum region far away from the black hole has the asymptotic form (51).

This suggests the following interpretation for the static instanton beyond the Nariai limit: it describes the thermal production of black holes of mass M [given by (11)] in an asymptotically de Sitter region. Initially, the throat of the black hole connects with a compact baby universe, but this pinches off as the black hole singularity develops [9]. The baby universe contains a bubble of the new phase in unstable equilibrium (see Fig. 3). If the bubble of the new phase collapses, the baby universe disappears into nothing. On the contrary, if the unstable bubble expands, it ends up generating an infinite region of the new vacuum phase, separated from an infinite region of the old vacuum phase by a domain wall in constant acceleration. At the center of the region of the old vacuum phase, there is also a black hole of mass M.

Let us now comment on the nucleation rate. According to Eq. (50), this is given by

$$\Gamma \sim e^{+S_{bh} - A(H_o^{-1})/4G} \sim e^{-\beta_o M + S_{bh}} e^{-A(r_+)/4G}. \qquad (GMH_o \ll 1) \qquad (52)$$

Here, S_{bh} is the black hole entropy. In the last step, we have used that for black holes of sufficiently low mass, the entropy of the cosmological horizon is smaller than the entropy of the original de Sitter metric by the amount $(\Delta S)_{horizon} = -\beta_o M$, as discussed in Section IV.

When we compare the previous result with Eq. (33), the last factor in Eq. (52) strikes us as rather unexpected. It seems to say that of all attempts at forming a black hole of mass M in a region with effective de Sitter temperature β_o, only a very small fraction given by $\exp[-A(r_+)/4G]$ succeed in forming a baby universe which hosts a bubble of the new vacuum
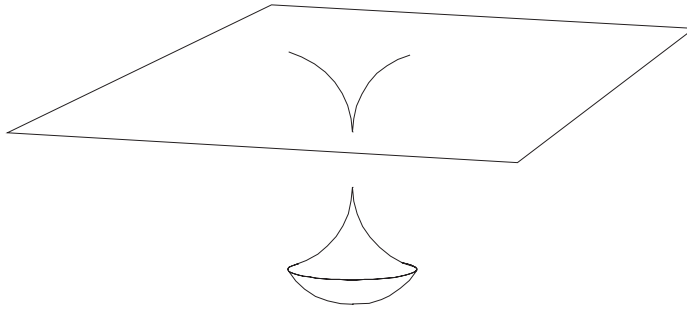
FIG. 3: A baby universe whith a bubble of the new phase, pinching off a large universe filled with the old phase.

phase. Perhaps this is not unreasonable, since a baby universe which is entirely filled with the old vacuum phase would have an entropy which is higher by the amount $+A(r_+)/4G$, relative to the entropy of the baby universe containing the static bubble. This suggests that most attempts should produce a baby universe of the old phase, without a bubble of the new phase. However, we should also keep in mind that the instanton representing this alledgedly more frequent process does not exist (the solution would contain two horizons at different temperatures, and hence the Euclidean section would have a conical singularity at one of them).

Mathematically, the factor $\exp[-A(r_+)/4G]$ arises because the instanton represented in Fig. 2 does not contain the cosmological horizon at r_+. The neighborhood of this horizon has been excised and replaced with the bubble of the new vacuum phase. This could mean that the interpretation given above for the instanton beyond the Nariai limit is not correct. In this interpretation, we are assuming the existence of an initial region, of size larger than the cosmological horizon, where the metric takes the approximate form (51). A cosmological horizon, and an asymptotic de Sitter region of the form (51), does develop if we let one of the unstable bubbles expand, but strictly speaking it is not present in the analytic continuation of the instanton. Clearly, the legitimacy of this interpretation deserves further investigation.

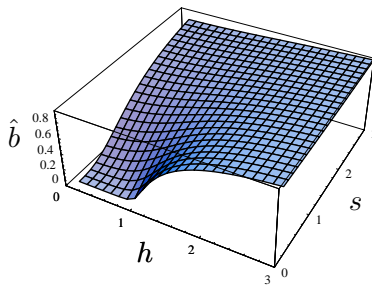## VI. COMPARISON WITH THE COLEMAN-DE LUCCIA ACTION

FIG. 4: The bounce action for the Coleman-De Luccia instantons. Here $\hat{b} = H_o^2 B_{\mathrm{CDL}}/8\pi^2 M_p^2$, $s = \sigma/2M_p^2 H_o$ and $h = H_i/H_o$. The action is finite also for upward jumps, which correspond to $h > 1$.

Let us now compare the action of the thermal instanton with that of the tunneling process

described by the Coleman-De Luccia (CDL) instanton. The latter is given by (see e.g. [25])

$$B_{CDL} = 12\pi^2 M_p^4 \left[\frac{1}{\Lambda_o}(1 - b\alpha_o) - \frac{1}{\Lambda_i}(1 - b\alpha_i) \right],$$ (53)

where

$$\alpha_{o,i} = \frac{\epsilon}{3\sigma} \mp \frac{\sigma}{4M_p^2},$$ (54)

and

$$b = \frac{1}{\sqrt{H_i^2 + \alpha_i^2}}.$$ (55)

The α's are related by $H_i^2 + \alpha_i^2 = H_o^2 + \alpha_o^2$. Using this relation and $H^2 = \Lambda/3M_p^2$, B_{CDL} can be written as

$$B_{CDL} = \frac{8\pi^2 M_p^2}{H_o^2} \frac{1}{2\sqrt{H_i^2 + \alpha_i^2}} \left[\frac{\sigma}{2M_p^2} - \frac{\epsilon}{3M_p^2} \left(\sqrt{H_i^2 + \alpha_i^2} - \alpha_i \right) \right].$$ (56)

The values of B for the static instanton and B_{CDL} are easily compared by noticing that both are of the form π/GH_o^2, times a function of the dimensionless parameters $s = \sigma/2M_p^2 H_o$ and $h = H_i/H_o$. In Fig. 4 we plot the action for the CDL case, and in Fig. 5 we plot the difference between the two actions. Note that the static instanton action is larger than the CDL action in the whole range of parameters.
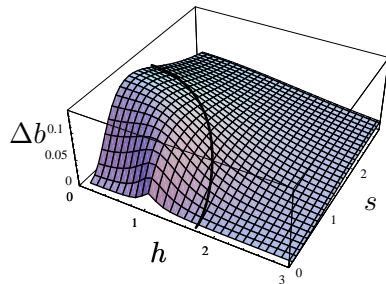

FIG. 5: The difference between the actions of the static and the Coleman-De Luccia instantons. Here $\Delta b = H_o^2(B - B_{CDL})/8\pi^2 M_p^2$, $s = \sigma/2M_p^2 H_o$ and $h = H_i/H_o$. The black line on the surface indicates the value σ_N for each value of H_i/H_o. Note that $\Delta b > 0$ in the whole range, and therefore the thermal activation process is always subdominant with respect to the Coleman-de Luccia tunneling process.

As we mentioned in Section IV, jumps to a vacuum with higher energy density are also allowed [21]. Note that for the case of upward jumps, $h^2 \gtrsim 1$, the actions become comparable, and in fact they are equal at the corner where $\sigma \to \sigma_N \to 0$ and $h^2 \to 3$ (see Figs. 5 and 6).

VII.   SEEDS OF THE NEW PHASE VS. REMNANTS OF THE OLD PHASE

In the interpretation which we have adopted so far, the static instanton represents the creation of pairs of critical bubbles of the new phase embedded in the false vacuum phase
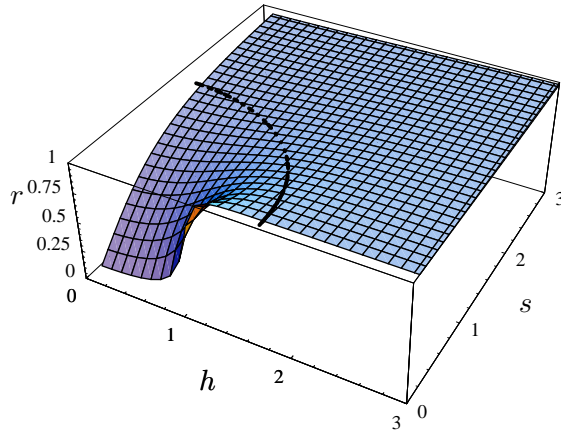
FIG. 6: The ratio $r = B_{CDL}/B$ between the actions of the Coleman-De Luccia and static instantons. As in Fig. 5, the black line on the surface indicates the value of σ_N. The two actions are comparable for $h^2 \gtrsim 1$, and become equal only at the corner where $\sigma \to \sigma_N \to 0$ and $h^2 \to 3$.
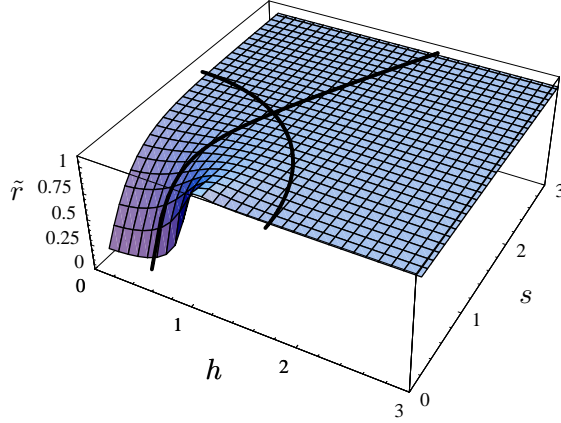

FIG. 7: The ratio $\tilde{r} = B_{CS}/B_{CR}$ between the bounce actions for the same transition between some initial vacuum and some final vacuum, where $h = H_{final}/H_{original}$, B_{CS} corresponds to the creation of a seed of the final vacuum and B_{CR} corresponds to the process which leaves a remnant of the original vacuum. The right and left boundary curves correspond to the Nariai limit for the creation of seeds and remnants respectively, $s = s_N$ and $s = \tilde{s}_N$ [see Eqs. (63) and (64)].

[12]. We may refer to this as the process of "pair creation of seeds" of the new phase. This process is analogous to pair creation of particles (or even topological defects such as monopoles [17]) by the expanding de Sitter background. As we showed in Section IV, when the nucleated objects are sufficiently light, the creation rate is simply proportional to the Boltzmann factor.

In Ref. [13], Gomberoff et al. suggested a rather different interpretation of the same solution. The process they considered involves a spherical bubble wall coming in from the cosmological horizon, sweeping away the false vacuum as it moves towards smaller radii, and replacing it with the true vacuum. The result of this process would also be a critical bubble in unstable equilibrium between expansion and collapse, but this time the bubble would be

a "remnant" of the old phase rather than the seed of the new phase. We shall thus refer to this process as " creation of remnants"[2]. Even if mathematically the Euclidean solution is the same as before, the interpretation and background subtractions are very different. As a consequence, the nucleation rate of such objects does *not* have the same simple Boltzmann suppression form as we found in (33) for light bubbles.

Suppose for definiteness a potential with two non-degenerate vacua, labeled by 1 and 2, with $V(1) > V(2) > 0$. The solution representing a downward jump which leaves a remnant of vacuum 1 surrounded by vacuum 2 is the same as the instanton for an upward jump caused by a seed of vacuum 1 which has been activated from vacuum 2. Similarly, the upward jump which leaves a remnant of vacuum 2 surrounded by vacuum 1 is related to downward jumps by activation of a seed of vacuum 2 from vacuum 1. Hence

$$S_{CR}^{\downarrow} = S_{CS}^{\uparrow}, \qquad S_{CR}^{\uparrow} = S_{CS}^{\downarrow}. \tag{57}$$

Here, the subindex CR stands for "creation of remnants", and S_{CR} denotes the action for the cosmological thermalon discussed by Gomberoff et al., while the subindex CS refers to "creation of seeds", and S_{CS} denotes the action given in (29). The arrows indicate whether we are considering an upward jump or a downward jump.

The bounce action is obtained by performing the relevant background subtractions

$$B_{CR}^{\downarrow} = S_{CS}^{\uparrow} - S(1), \qquad B_{CR}^{\uparrow} = S_{CS}^{\downarrow} - S(2). \tag{58}$$

Here $S(1)$ and $S(2)$ are the background actions, given by Eq. (29), with r_+ replaced by the corresponding de Sitter radii H_1^{-1} and H_2^{-1} respectively. Before proceeding, we should stress that since the instantons we are considering are static and compact, then according to the discussion in Section III the bounce actions are always given by [3]

$$B_i = -\Delta A/4G = -\Delta S \tag{59}$$

where ΔA is the change in the area of the horizon and ΔS is the change in the entropy.

---

[2] Gomberoff et al. used the term "cosmological thermalon" for the process of creation of remnants. "Thermalon" may indeed be a better word than "instanton" for describing the static Euclidean solutions. However, this denomination seems equally appropriate for the process of creation of seeds, so to avoid confusion we shall simply refer to creation of seeds or remnants.

[3] In fact, Ref. [13] considered a slightly different setting, where a membrane is coupled to a three form gauge field A_3 and to gravity. The term which represents the interaction of the membrane with the gauge field takes the form $q \int A_3$, where q is the membrane charge, and the integral is over the membrane worldsheet. It was argued in [13] that A_3 is discontinuous accross the membrane, and a somewhat heuristic prescription was given to compute the contribution of $q \int A_3$ to the action and to perform the background subtraction. The result of this procedure, however, differs from Eq. (59). Here, we shall not try to elucidate the reason for this discrepancy. We note, however, that the on-shell Euclidean action for the system of a membrane coupled to A_3 and to gravity, and with proper inclusion of boundary terms [23], can be shown to be the same as the action we have taken as our starting point (25) with ρ_V replaced by $F^2/2$, where $F = dA_3$ is the field strength [see e.g. Eq. (6.1) in Ref. [23]]. Hence, the results of the present paper, which are in principle valid for the case of vacuum decay in field theory, may as well be valid for the case of the brane coupled to the antisymmetric tensor field.

It is clear from (58) that

$$B_{CR}^{\uparrow} - B_{CDL}^{\uparrow} = B_{CS}^{\downarrow} - B_{CDL}^{\downarrow} > 0, \qquad (60)$$
$$B_{CR}^{\downarrow} - B_{CDL}^{\downarrow} = B_{CS}^{\uparrow} - B_{CDL}^{\uparrow} > 0. \qquad (61)$$

Here, we have used the fact that $B_{CDL}^{\uparrow} + S(2) = B_{CDL}^{\downarrow} + S(1)$, since the CDL instanton solution is the same for upward and for downward jumps, and all that changes is the background subtraction [21]. The inequalities above come from the fact that B_{CS} is always larger than B_{CDL}, as shown in the previous Section. It follows that B_{CR} is also always larger than B_{CDL}, and so the creation of remnants is also subdominant with respect to the tunneling process represented by the CDL instanton.

Finally, we may ask which of the two processes is more important, the creation of seeds or the creation of remnants. From (57) we have

$$B_{CR}^{\uparrow} - B_{CS}^{\uparrow} = B_{CS}^{\downarrow} - B_{CR}^{\downarrow}. \qquad (62)$$

Hence, if one of the channels is dominant for upward jumps, then it means that the other process is dominant for downward jumps. Fig. 7 shows the ratio $\tilde{r} = B_{CS}/B_{CR}$. Note that if $h < 1$, corresponding to downward jumps, then the process of pair creation of seeds is much more likely than the process of pair creation of remnants. On the other hand, for $h > 1$, corresponding to upward jumps, the ratio of the bounce actions is very close to one [although, from (62), the frequency of upward jumps through creation of remnants outweighs that of upward jumps through creation of seeds by the same factor as downward jumps through creation of seeds outweight downward jumps through creation of remnants.]

Finally, let us recall that the Nariai limit corresponds to $\sigma_N = 2M_p^2(3H_o^2 - H_i^2)^{1/2}$ [see Eq. (12)]. Here the indices i and o stand for the inside and the outside of the bubble. For pair creation of seeds, outside and inside correspond to the original vacuum and the final vacuum respectively $H_o = H_{original}$ and $H_i = H_{final}$. In the dimensionless variables $s = \sigma/(2M_p^2 H_{original})$ and $h = H_{final}/H_{original}$, the Nariai curve corresponds to

$$s_N^2 = 3 - h^2. \qquad (63)$$

For pair creation of remnants with the same initial and final states as the seeds, we have $H_o = H_{final}$ and $H_i = H_{original}$, and the Nariai limit corresponds to

$$\tilde{s}_N^2 = 3h^2 - 1. \qquad (64)$$

The Nariai curves $s = s_N$ and $s = \tilde{s}_N$, corresponding to the circle (63) and the hyperbola (64) are also plotted in Fig. 7.

VIII.   SUMMARY AND CONCLUSIONS

De Sitter vacua are believed to be metastable at best (see e.g. [24] for a recent discussion). It is well known that vacuum transitions from a metastable vacuum can proceed through quantum tunneling, which is described by the Coleman-de Luccia instanton. This process can take us to lower energy vacua, but also to other de Sitter vacua with a higher vacuum energy density [21].

Here, we have investigated an alternative process, by which critical bubbles of the new phase can be pair produced. This process is the analog of thermal activation in flat space. The mass M and radius R of the "seeds" of the new phase are given by Eqs. (10) and (11), in terms of the initial and final vacuum energies, $\rho_o = 3M_P^2 H_o^2$ and $\rho_i = 3M_P^2 H_i^2$, and of the tension σ of the wall separating both phases.

For $\sigma^2 < 4M_p^4(3H_o^2 - H_i^2)$, the geometry of the critical bubbles is the following (see Fig. 1). Outside the bubble, the metric is Schwarzschild-de Sitter, and has a cosmological horizon. The black hole horizon is not present, since we are matching to an interior solution at some $R > r_s$, where R is the bubble radius and r_s is the radius of the would be black hole horizon. Inside the bubble, the metric is pure de Sitter with curvature radius H_i^{-1}. For $GMH_o \ll 1$, the nucleation rate is proportional to the Boltzmann factor

$$\Gamma \sim e^{-\beta_o M}, \tag{65}$$

as would be expected from simple thermodinamical arguments. Here $\beta_o = 2\pi/H_o$ is the inverse de Sitter temperature of the old vacuum phase.

For $\sigma^2 = 4M_p^4(3H_o^2 - H_i^2)$ the metric outside of the bubble corresponds to the Nariai limit of the Schwarzschild-de Sitter solution, for which the black hole and cosmological horizons have the same size. Beyond the Nariai limit, i.e. for $\sigma^2 > 4M_p^4(3H_o^2 - H_i^2)$, the asymptotic form of the solution changes quite drastically [13]. The static solution with a pair of critical bubbles has a black hole horizon instead of a cosmological horizon. The interpretation of such solution is less clear than in the case $\sigma^2 < 4M_p^4(3H_o^2 - H_i^2)$, but we have argued that it may correspond to the creation of a baby universe containing a bubble of the new phase. The nucleation rate is formally given by (52), and does not have the simple form (65) even in the case when the mass M is small (here, M is the mass of the black hole connecting the asymptotic region of the old phase with the baby universe).

We have compared the process of thermal activation of seeds to an alternative process recently suggested by Gomberoff et al. [13], by which most of space would suddenly jump to the new vacuum phase, leaving only a pair of critical bubbles as remnants of the old phase. These could subsequently collapse into black holes, with the net result that the vacuum "dark" energy, is transformed into cold "dark matter" in the form of black holes. We find that for downward jumps, this process is subdominant with respect to the thermal activation of seeds of the new vacuum. For upward jumps, the bounce actions are comparable, and in fact the creation of remnants may be slightly favoured with respect to the creation of seeds (although when we are going up in energy we are not transforming dark energy into dark matter, but simply increasing both of them!).

Also, we have compared the rate of nucleation of critical bubbles by thermal activation with the rate of bubble nucleation by quantum tunneling, described by the Coleman-de Luccia (CDL) instanton. The CDL instanton always has a lower bounce action than the process of thermal activation of seeds or remnants. Thus, even if thermal activation is possible, it appears that jumps between neighboring vacua will be more frequent through quantum tunneling. For the case of upward jumps, however, the corresponding actions are comparable (see Fig. 6). Since the action for thermal activation is higher than that for tunneling, one should ask whether there are any situations where the former process may nevertheless be relevant. Note that if the bubble wall carries some internal degrees of freedom, their entropy will be accounted for in the prefactor which accompanies the leading expression e^{-B} for the nucleation rate. It is clear from Eq. (46) that the temperature the bubble wall can be very high if the wall tension and the vacuum energies in the two phases

are suitably adjusted. Hence the entropy of the internal degrees of freedom can be very high, making up perhaps for the difference in actions. Investigation of this possibility is left for further research. [4]

Acknowledgements

------

[1] S. Coleman, Phys. Rev. **D15**, 2929 (1977).

[2] S. R. Coleman, V. Glaser and A. Martin, Commun. Math. Phys. **58** (1978) 211.

[3] A. D. Linde, Nucl. Phys. B **216** (1983) 421 [Erratum-ibid. B **223** (1983) 544].

[4] S. Coleman and F. De Luccia, Phys. Rev. **D21**, 3305 (1980); J. D. Brown and C. Teitelboim, Phys. Lett. **B195**, 177, (1987); Nucl. Phys. **297**, 787 (1988).

[5] J. F. Donoghue, arXiv:hep-th/0310203.

[6] See e.g. A. Megevand, arXiv:hep-ph/0312305, and references therein.

[7] A. H. Guth and E. J. Weinberg, Phys. Rev. D **23** (1981) 876; Nucl. Phys. B **212** (1983) 321.

[8] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977).

[9] K. Sato, M. Sasaki, H. Kodama and K. i. Maeda, Prog. Theor. Phys. **65** (1981) 1443; H. Kodama, M. Sasaki, K. Sato and K. i. Maeda, Prog. Theor. Phys. **66** (1981) 2052; K. Sato, H. Kodama, M. Sasaki and K. i. Maeda, Phys. Lett. B **108** (1982) 103 ; H. Kodama, M. Sasaki and K. Sato, Prog. Theor. Phys. **68** (1982) 1979; S. K. Blau, E. I. Guendelman and A. H. Guth, Phys. Rev. D **35** (1987) 1747; E. Farhi and A. H. Guth, Phys. Lett. B **183** (1987) 149; E. Farhi, A. H. Guth and J. Guven, Nucl. Phys. B **339** (1990) 417.

[10] V. A. Berezin, V. A. Kuzmin and I. I. Tkachev, Phys. Scripta **T36** (1991) 269.

[11] D. A. Samuel and W. A. Hiscock, Phys. Rev. D **44** (1991) 3052.

[12] J. Garriga and A. Megevand, arXiv:hep-th/0310211. Phys. Rev. D. (in press).

[13] A. Gomberoff, M. Henneaux, C. Teitelboim and F. Wilczek, "Thermal decay of the cosmological constant into black holes," arXiv:hep-th/0311011.

------

[4] In Ref. [12], we speculated that thermal activation may be relevant for the process of multiple brane nucleation. As noted in [25], coincident branes carry a number of degrees of freedom which grows nonlinearly with the number of branes. Because of that, the entropy factors due to the fields living on the branes may greatly enhance the nucleation rates, and it may be more probable to nucleate a bubble bounded by a whole stack of branes, than a bubble bounded by a single brane. Note, however, that in four dimensions (and after the dilaton is stabilized) the interactions amongst 2-branes are repulsive. Because of that, the CDL instanton for multiple brane nucleation may not exist, since the stack of branes does not hold together. On the other hand, at sufficiently high temperature, the branes may attract each other because of thermal symmetry restoration. Hence, it is conceivable that the CDL instanton may not exist while the thermal instanton does [12]. This possibility seems rather exotic, but it may well be realized in certain regions of parameter space.

[14] W. Israel, Nuovo Cimento **44B**, 1 (1966), and correction in **48B**, 463 (1967).

[15] P. Ginsparg and M. J. Perry, Nucl. Phys. B **222** (1983) 245.

[16] H. Nariai, Science Reports of the Tohoku Univ. 35 (1951) 62.

[17] R. Basu, A. Guth and A. Vilenkin, Phys. Rev. **D44**, 340 (1991).

[18] M. Banados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **72** (1994) 957 [arXiv:gr-qc/9309026].

[19] S. W. Hawking and G. T. Horowitz, Class. Quant. Grav. **13** (1996) 1487 [arXiv:gr-qc/9501014].

[20] C. Teitelboim, "Gravitational Thermodynamics of Schwarzschild-de Sitter Space," arXiv:hep-th/0203258.

[21] K. M. Lee and E. J. Weinberg, Phys. Rev. D **36** (1987) 1088; J. Garriga, Phys. Rev. D **49** (1994) 6327 [arXiv:hep-ph/9308280]; Phys. Rev. D **49** (1994) 6343; J. Garriga and A. Vilenkin, Phys. Rev. D **57** (1998) 2230 [arXiv:astro-ph/9707292].

[22] S. W. Hawking and I. G. Moss, Nucl. Phys. B **224**, 180 (1983).

[23] J. D. Brown and C. Teitelboim, Nucl. Phys. B **297** (1988) 787.

[24] L. Susskind, "The anthropic landscape of string theory", hep-th/0302219.

[25] J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, Nucl. Phys. **B602**, 307 (2001).